

Exercise 66

Find the equation of the line that passes through the following points: $(2a, b)$ and $(a, b + 1)$

Solution

The general equation for a line is

$$Y = MX + B.$$

The first point says that when the input is $X = 2a$, the output is $Y = b$.

$$b = M(2a) + B$$

The second point says that when the input is $X = a$, the output is $Y = b + 1$.

$$b + 1 = M(a) + B$$

This is a system of two equations for two unknowns that can be solved for.

$$\begin{cases} 2aM + B = b \\ aM + B = b + 1 \end{cases}$$

Subtract the respective sides of these equations to eliminate B .

$$2aM - aM = b - (b + 1) \quad \rightarrow \quad aM = -1 \quad \rightarrow \quad M = -\frac{1}{a}$$

Multiply both sides of the second equation by -2

$$\begin{cases} 2aM + B = b \\ -2aM - 2B = -2b - 2 \end{cases}$$

and then add the respective sides to eliminate M .

$$B + (-2B) = b + (-2b - 2) \quad \rightarrow \quad -B = -b - 2 \quad \rightarrow \quad B = b + 2$$

Now that M and B are solved for, the equation of the line is known.

$$Y = -\frac{1}{a}X + (b + 2)$$